Friction in Curling Game

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Abstract: Dynamics of a curling stone is considered. Challenge for scientists is the strange behavior of a stone: its trajectory is bent aside, opposite to the predicted. Moreover, none of usual model of friction explains this effect quantitatively. We propose a model of anisotropic friction which agrees with experimental data.

Keywords: Friction, Mechanical systems, Contact resistance, Dynamic modeling

1. INTRODUCTION

The curling game has arisen in Scotland, one of the oldest curling stones was dated 1511 (Fig. 1). In 1988 curling has been included in the program of the Olympic games. Modern curling stones have the cylindrical form with a diameter 29 cm, height 11.4 cm and mass 20 kg. They are made of a special granite which is extracted in Scotland and the Wales. The bottom part of an apparatus is bent, and it leans against ice a ring platform with a diameter 12.5 cm and width 4-5 mm (Fig. 2).

Fig. 1. The first curling stone.

In the top part of the apparatus there is a handle with which help the sportsman gives it rotation. The given circumstance is rather important for two reasons. First, rotation reduces total sliding friction. Secondly, the stone trajectory is bent that allows to solve various tactical problems. This curvature is usually explained by non-uniformity of distribution of normal loading and/or dependence of coefficient of friction μ on the relative sliding velocity v. It is easy to show (see e.g. Penner (2001)) that in the case $\mu = \text{const}$ the trajectory of a cylinder, rotating counter-clockwise, deviates to the right. Actually, the trajectory deviates to the left!

Fig.2. Modern apparatuses.

Johnston (1981) suggested that function $\mu(p)$ is decreasing. Penner (2001) established experimental dependence

$$\mu = 0.008v^{-1/2}$$

and noted that certain adhesion effects may take place in the slow (left) part of contact area. He has considered various models of frictions and comes to the conclusion that none of them is satisfactory.

Shegelski (2000) supposed that μ depends on the angle θ between radius-vector of given point of the contact ring and velocity vector of the center of mass, such that

$$\mu(\theta) = \mu_0(1 - f_0\cos\theta)$$

where μ_0 and f_0 are some constants. In his calculations $p = \text{const}$.

Denny (2002) has assumed that contact conditions change in the course of stone movement. The snow crumb sticks to the forward part of the contact ring, this leads to reduction of friction in the given points. At the initial moment of movement a surface of a ring is clear. Through small time the forward part becomes “polluted”. Owing to rotation the polluted part with the reduced coefficient of a friction gradually extends on all ring. Up to this moment, asymmetry of frictional forces leads to the net force, directed to the left.

Jensen and Shegelski (2004) have proposed a semi-phenomenological model of dynamics of the curling stone. Being based on experimental data, they concluded that the frictional force acting upon each segment of the curling is directed opposite to the motion relative to this thin liquid film and not relative to the underlying fixed ice surface. They did not deduce a formula of the local law of friction, but stated that the net frictional force has the constant transverse component and the tangent component which is a power function of sliding velocity.

The listed models to some extent can serve for the description of dynamics of a curling stone. Their common fault consists in discrepancy to laws of dynamics: a realistic model of friction must account correlation between used law of friction and distribution of normal load.

The paper is organized as follows. In section 2 the idea of dynamical consistence is introduced. Section 3 is devoted to the comparative analysis of Coulomb friction with various functions $\mu(v, p)$. In Section 4 a model of anisotropic friction is suggested which agrees with experimental data. Section 5 contains some concluding remarks.

2. IDEA OF DYNAMICAL CONSISTENCE

Consider a heavy rigid body with flat basement moving upon a horizontal plane. Equation of motion We write down equations of motion in Newton – Euler form:

$$m\left(d\vec{v}/dt\right) = \vec{N} - mg\vec{k} + \vec{T},\ \ d(J\vec{\Omega})/dt = \vec{M}_N + \vec{M}_T\,, \quad (1)$$

where $\vec{v}$ is velocity of the center of mass G, $\vec{\Omega}$ is angular velocity of the body, m is the mass, J is the central tensor of inertia, $\vec{N}$ and $\vec{M}_N$ are net normal reactions and their moment relative to G, $\vec{T}$ and $\vec{M}_T$ are net friction force and their moment, $\vec{k}$ is the ort directed upwards.

Let $n(A)$ and $\vec{t}(A)$ be the normal stress and friction force at a point A, which belongs to the contact area D. Then

$$\begin{aligned}&\vec{N} = \vec{k}\iint n(A)dS,\ \ \vec{T} = \iint \vec{t}(A)dS,\\ &\vec{M}_N = \iint\left(\vec{r}(A)\times\vec{k}\right)n(A)dS,\ \vec{M}_T = \iint\left(\vec{r}(A)\times\vec{t}(A)\right)dS\end{aligned} \quad (2)$$

(the integrals are calculated on area D). To calculate friction forces, a law of friction should be specified. We adopt the following general form:

$$\vec{t}(A) = -\mu\left(n(A), v(A)\right)n(A)\vec{e}\left(\vec{v}(A)\right), \quad (3)$$

where the coefficient of friction μ depends on the normal stress and sliding velocity and direction of friction force $\vec{e}$ depends on the direction of sliding velocity.

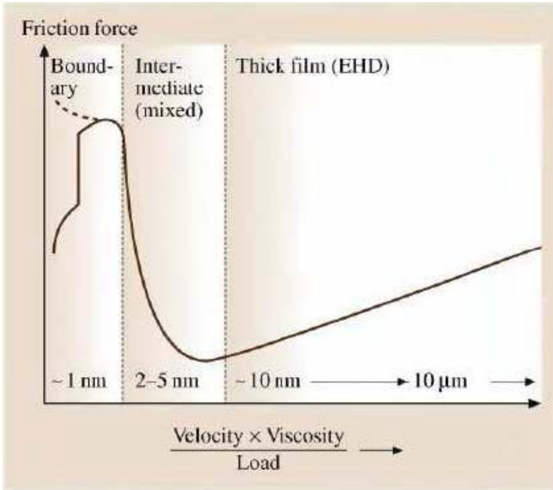

Fig.3. Stribeck curve (from Bhushan, 2004).

For standard Coulomb friction we have

$$\vec{e}(\vec{v}) = \vec{v}/|\vec{v}| \quad (4)$$

and $\mu = \text{const}$. In the presence of a liq1uid film which appears due to melting of ice the dependence of μ on v and/or p should be accounted. A typical experimental Stribeck curve is presented in Fig.3, where on an axis of abscisses it is counted Hersey-Gümbel number, which is proportional to v/n and equals the thickness of liquid film.

Further complication is model of anisotropic friction where

$$\vec{t}(A) = -n(A)B(A)\left(\vec{v}(A)\right)/|\vec{v}(A)| \quad (5)$$

with tensor of friction $B(A)$ being a positive 2×2 matrix which depends in general on $v(A)$ and $n(A)$. In particular, if $B(A)$ is a scalar matrix (i.e. diagonal with equal elements $b_{11} = b_{22} = \mu$), then formula (5) coincides with (3). In general friction force (5) depends on direction of sliding and is not collinear to it. At intuitive level, such friction can be interpreted as resistance of a wavy surface. To move across a wave it is more difficult, than along its front; at movement obliquely force of a friction forms with front an angle, which is more than an angle of vector of speed with front.

Since the body moves parallel the horizontal plane, we have

$$\left(\vec{v},\vec{k}\right) = 0,\ \ \vec{\Omega} = \omega\vec{k} \Rightarrow \vec{\Omega}\times\vec{k} = 0\,, \quad (6)$$

where ω is a scalar. Substituting (6) into (1), we obtain

$$N = mg,\ \ J\vec{k}(d\omega/dt) + \omega\vec{k}\times(J\vec{k}) = \vec{M}_N + \vec{M}_T\,. \quad (7)$$

In case of axial symmetrical bodies (such as curling stones) vector $J\vec{k}$ is collinear to $\vec{k}$, therefore, $\vec{k}\times(J\vec{k}) = 0$. Thus, conditions (7) can be presented in the form

$$N = mg,\ \left(\vec{M}_N + \vec{M}_T\right)\times\vec{k} = 0 \quad (8)$$

Equations (8) are called *conditions of dynamical consistency* (Ivanov, 2009). The following result has been proven by the author (Ivanov, 2011; Ivanov, Shuvalov, 2012).

Proposition 1. Suppose that for given body $\vec{k}\times(J\vec{k}) = 0$. Then *for any friction law* (3) vectors $\vec{T}$ and $\vec{M}_N$ are orthogonal, and vectors $\vec{T}$, $\vec{M}_N$, and $\vec{k}$ form the right triple (Fig.4).

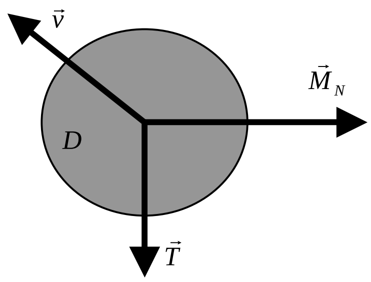

Fig.4. Net friction force and net normal moment

As soon as a distribution of the normal load $n(A)$ is given, we can determine friction by formula (3) and then calculate integrals (2). However, conditions (8) can be violated. Since they are equivalent to three scalar equations, one should have at hands a three-parameter family of admissible distributions $n(A)$. These parameters are to be adjusted so to satisfy the conditions. The most simple and popular model is described by the following linear formula

$$n(A) = n_0 + n_1\xi + n_2\eta\ , \quad (9)$$

where ξ and η are co-ordinates of vector $G'A$, where G' is projection of point G to the ground.

3. ANALYSIS OF ISOTROPIC FRICTION

First consider the case of isotropic friction (3), (4). The following dynamical properties were established as the Corollaries of the Proposition 1 in Ivanov (2011), Ivanov, Shuvalov (2012) for an axially symmetric rigid body with ring base. In the following statements, it is suggested that the normal load (weight) is distributed linearly in accordance with (9).

Proposition 2. In case $\mu = \text{const}$ the trajectory deviates to the right (for anti-clockwise rotation of the body), i.e. contrarily to the motion of the culling stone.

Proposition 3. If coefficient $\mu(A)$ is any function of $v(A)$, and spinning is not too fast (such that the instant centre of velocities lies outside the base), then the trajectory deviates to the right. For fast spinning and ring contact area, deviation to the left is possible.

Remark. It can be shown that deviation to the left occurs provided $n_1 > 0, n_2 < 0$ (y - axis being directed along the velocity vector of the center of mass). In this case, the rotation is slowed down while sliding is accelerated. Such behavior is not observed in the game of curling.

Let us pass now to consideration of case where the coefficient of friction is a function of Hersey – Gümbel number

$$\mu(A) = \mu\left(v(A)/n(A)\right)\ . \quad (10)$$

Approximate dependence (10) is shown in Fig.2. We can see that in each of the regions of mixed friction and of elasto-hydrodynamic (EHD) friction this function may be approximated by a linear function

$$\mu = \mu_0 + \mu_1 v(A)/n(A) \quad (11)$$

Proposition 4. (i) If $\mu_0 > 0, \mu_1 > 0$, then friction (11) leads to the deviation to the right.
(ii) In case $\mu_0 > 0, \mu_1 < 0$ the curvature is similar to the friction, being considered in Proposition 3. That is, for small rates of rotation the trajectory deviates to the right. For fast spinning deviation to the left is possible with simultaneous acceleration of sliding velocity.
(iii) If $\mu_0 < 0, \mu_1 > 0$, the trajectory deviates to the left with simultaneous deceleration of sliding velocity.

Remark. The first and the third regions in Fig.3 (boundary and EHD friction) can be described by cases (i) or (iii). The second region (mixed friction) corresponds to case (ii). Therefore, observed effect of curling can be explained *qualitatively* by formula (11) provided the extension of linear part of graph in Fig. 3 intersects the abscissa axis in positive region.

Now we study the following problem: if any *isotropic* friction law (3), (4) provides *quantitative* agreement with experimental data. For the comparative analysis, we use the following approximate expressions, which have been derived by Jensen and Shegelski (2004):

$$x(t) = -\left(0.004t^2 - 0.185t + 2.98\left(1-\left(1-t/23\right)^{1.43}\right)\right)$$
$$y(t) = 25*\left(1-\left(1-t/23\right)^{1.897}\right),\ \varphi(t) = 16.58*\left(1-\left(1-t/24\right)^{1.455}\right) \quad (12)$$

where t denotes duration of motion, φ is angle of rotation of the stone, x and y are the co-ordinates of the center mass G in fixed frame (y axis is directed along the curling sheet). According to formulas (12),

$$x(0) = y(0) = 0\ ,\ dx/dt = 0\ ,\ dy/dt = 2\,\text{m/s},\ \omega = 1s^{-1}\ . \quad (13)$$

For the time $t = 23$ the stone covers 25 m in forward direction and deviates to the left on 0.84 m. In this time it makes 2.6 turns.

Proposition 5. The trajectory (12) is caused by a friction force which can not be described by any isotropic law (3), (4).

Proof. Friction force $\vec{T} = \left(T_x, T_y\right)$ can be determined from the first equation (1) by differentiation of formulas (12):

$$\begin{aligned} T_x &= md^2x/dt^2 = -m\left(0.008 + 0.0035\left(1-t/23\right)^{-0.57}\right) \\ T_y &= md^2y/dt^2 = -0.08m\left(1-t/23\right)^{-0.013} \end{aligned}. \quad (14)$$

Due to (14), for any t we have $T_x \geq 0.1T_y$. If formulas (3) and (4) are valid, then according to relations (2) in some points of the contact ring the inequality

$$dx(A)/dt \geq 0.01dy(A)/dt \ . \qquad (15)$$

By Euler formula,

$$dx(A)/dt = dx/dt - \omega\eta, \ \ dy(A)/dt = dy/dt + \omega\xi \, . \qquad (16)$$

Differentiating (12) and substituting the results into (16), we obtain

$$dx(A)/dy(A) \leq 0.032(1-t/24)^{0.455}(1-t/23)^{-0.897} \leq 0.09 \quad (17)$$

for any $t \leq 20$ s. Therefore, inequality (15) is not satisfied. This contradiction shows that formulas (3), (4) are not applicable to the trajectory (12).

4. DETERMINATION OF ANISOTROPIC FRICTION

As was shown in the previous Section, no isotropic friction law is in satisfactory agreement with experimental data on dynamics of the curling stone. A physical reason for anisotropy (i.e. dependence of the friction force on the sliding direction) is the presence of liquid film, which appears at ice melting. Further we will neglect width of the contact ring.

Consider a point $A \in D$, then its co-ordinates with respect to center of mass are $\xi = R\cos\alpha, \eta = R\sin\alpha$, where α is the polar angle and $R = 0.065$ m is radius of the ring. There is a water film inside the ring, since these points have been subject to thermal influence owing to a friction at stone movement on ice. However, there is no film outside the ring. Further, in case where the stone just rotates without moving forward, vector $\vec{v}(A)$ is tangent to the ring. Irrespective of a rotation direction, similar liquid film appears. Thus, there are three different coefficients of friction (see Fig.5):

(i) μ_1 – for sliding along outward normal $\vec{n} = (\cos\alpha, \sin\alpha)$;

(ii) μ_2 – for sliding along inward normal $-\vec{n}$;

(iii) μ_3 – for sliding along tangent $\vec{\tau}(A) = (-\sin\alpha, \cos\alpha)$, $\vec{v}(A)$ is directed outward;

(iv) μ_4 – for sliding along tangent $\vec{\tau}(A)$, $\vec{v}(A)$ is directed inward.

Such situation is more general than formula (5) if $\mu_1 \neq \mu_2$, since

$$\begin{aligned} &B(A)\left(\vec{n}(A)\right) = \mu_1\vec{n}(A), \ B(A)\left(-\vec{n}(A)\right) = -\mu_2\vec{n}(A), \\ &B(A)\left(\vec{\tau}(A)\right) = \mu_{3,4}\vec{\tau}(A). \end{aligned} \qquad (18)$$

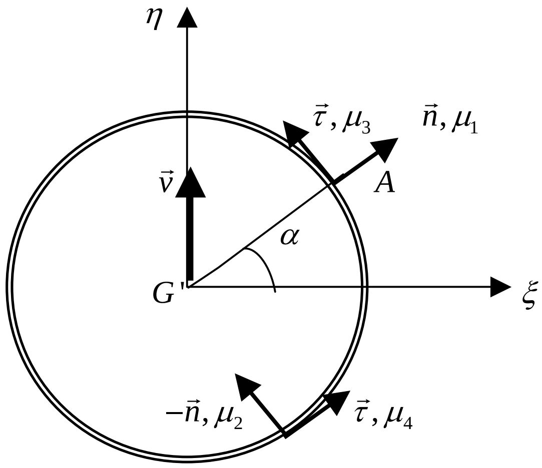

Fig.5. Characteristics of anisotropic friction

To calculate friction force, one should at first present $\vec{v}(A)$ in the form

$$\vec{v}(A) = a\vec{n}(A) + b\vec{\tau}(A) \qquad (19)$$

and then apply relations (18). Therefore,

$$B(A)\left(\vec{v}(A)\right) = \begin{cases} \mu_1 a\vec{n}(A) + \mu_3 b\vec{\tau}(A), \ if \ \ a > 0 \\ \mu_2 a\vec{n}(A) + \mu_4 b\vec{\tau}(A), \ if \ \ a < 0 \end{cases} \qquad (20)$$

Substituting (20) into (5), we obtain four-parameter friction law; after calculations by formulas (2) and comparison with (14) we can evaluate these parameters.

We adopt a simplified model where $n(A) = \gamma = mg/(2\pi R)$, i.e. the normal load is distributed uniformly in the contact region (thin ring). Since the coefficient of friction in curling is of order 0.01, such approximation leads to an error about 1%. On account of (16) and (19), we obtain

$$\vec{v}(A) = v\vec{n}(A)\sin\alpha + v\vec{\tau}(A)(\varepsilon + \cos\alpha), \quad \varepsilon = R\omega/v \, . \quad (21)$$

Application of formulas (5), (20), (21) in the forward half-circle $\alpha \in (0, \pi)$ leads to the relation

$$\begin{aligned} \vec{t}(A) = -\gamma((\mu_1 - \mu_3)\sin\alpha\cos\alpha - \varepsilon\mu_3\sin\alpha, \ \mu_1\sin^2\alpha + \\ + \mu_3\cos^2\alpha + \varepsilon\mu_3\cos\alpha)\left(1 + 2\varepsilon\cos\alpha + \varepsilon^2\right)^{-1/2} \end{aligned} \qquad (22)$$

In the points of backward half-circle $\alpha \in (-\pi, 0)$ the same relation (22) is applicable after replacing of μ_1 and μ_3 by μ_2 and μ_4 correspondingly.

According with (12), $\varepsilon \leq 0.1$ for $t \leq 20$ s. Hence, at calculation of integrals (2) we can consider number ε as small and account main terms only. Approximate computation lead to the following estimates:

$$\begin{aligned} T_\xi &= \varepsilon mg(\mu_1 - \mu_2 + 2\mu_3 - 2\mu_4)/(3\pi), \\ T_\eta &= -mg(\mu_1 + \mu_2 + \mu_3 + \mu_4)/4, \end{aligned} \qquad (23)$$

$$M_T = -\varepsilon mgR\left(\mu_3 + \mu_4\right)/4 \,,$$

where T_ξ and T_η are projection of net friction force to the related axes of moving frame $G'\xi\eta$. Let β be the angle between $G'x$ and $G'\xi$, then equations (1) can be written in the form

$$\begin{aligned}
&\ddot{x}\cos\beta - \ddot{y}\sin\beta = \varepsilon g(\mu_1 - \mu_2 + 2\mu_3 - 2\mu_4)/(3\pi),\\
&\ddot{x}\sin\beta + \ddot{y}\cos\beta = -g(\mu_1 + \mu_2 + \mu_3 + \mu_4)/4, \qquad (24)\\
&\rho^2\ddot{\varphi} = -\varepsilon gR(\mu_3 + \mu_4)/4
\end{aligned}$$

where the accelerations $\ddot{x}, \ddot{y}$ are defined by (14), $\rho = 0.1\,\text{m}$ is radius of gyration of the stone; angular acceleration $\ddot{\varphi}$ can be determined by differentiation of the third formula (12).

At each instant t, system of three equations (24) is linear with respect to unknowns μ_j $(j = 1,\ldots,4)$. The solution is not solely, but we can find uniquely some three independent combinations, say $l_1 = \mu_1 + \mu_2$, $l_2 = \mu_3 + \mu_4$, and $l_3 = \mu_1 + 2\mu_3$. The numerical analysis has been performed which showed that for all $t \le 20\,\text{s}$ values l_1 and l_3 are negative. Therefore, formula (20) is inapplicable to the explanation of observed dynamics.

Another model of anisotropic friction was suggested by Ivanov, Shuvalov (2012): if

$$\vec{v}(A) = a\vec{i} + b\vec{j} \,, \qquad (25)$$

where $\vec{i}$ and $\vec{j}$ are orts of axes $G'\xi$ and $G'\eta$, then

$$B(A)\left(\vec{v}(A)\right) = \begin{cases} \mu_{1\xi}a\vec{i} + \mu_{1\eta}b\vec{j}, \text{ if } \eta(A) > 0 \\ \mu_{2\xi}a\vec{i} + \mu_{2\eta}b\vec{j}, \text{ if } \eta(A) < 0 \end{cases} \qquad (26)$$

In this case similar to (24) we obtain the following systems:

$$\begin{aligned}
&\ddot{x}\cos\beta - \ddot{y}\sin\beta = \varepsilon g(\mu_{1\xi} - \mu_{2\xi})/\pi,\\
&\ddot{x}\sin\beta + \ddot{y}\cos\beta = -g(\mu_{1\eta} + \mu_{2\eta})/2, \qquad (27)\\
&\rho^2\ddot{\varphi} = -\varepsilon gR(\mu_{1\xi} + \mu_{2\xi})/4
\end{aligned}$$

Calculations were conducted for $R = 0.054\,\text{m}$. A good agreement with data (12) are attained for

$$\mu_{1\xi} = 0.0018,\ \mu_{2\xi} = 0.058,\ \mu_{1\eta} + \mu_{2\eta} = 0.018\,. \qquad (28)$$

Note that coefficients $\mu_{1\eta}$ and $\mu_{2\eta}$ in this model of friction can not be defined uniquely and that friction in transversal direction is about three times more than friction in tangent direction.

5. CONCLUSIONS

It is shown that some of models of isotropic friction explain the deviation to the left of a curling stone rotating counter-clockwise. However, none of them provides satisfactory quantitative agreement with experimental data. As concern to anisotropic friction, there is a considerable quantity of variants in a model choice. We presented only two simplest four-parameter models and showed that one of them is inadequate while another is sufficient to describe observed dynamics. There still exist a vast field for elaboration more comprehensive models on the base of analysis more experimental data.

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