Describing Human Emotions Through Mathematical Modelling ⋆

Kim Hartmann * **Ingo Siegert** * **Stefan Glüge** *
Andreas Wendemuth * **Michael Kotzyba** ** **Barbara Deml** **

* *Faculty of Electrical Engineering and Information Technology, Institute for Electronics, Signal Processing and Communications, Otto-von-Guericke-University, 39106 Magdeburg, Germany (e-mail: firstname.lastname@ovgu.de)*
** *Faculty of Mechanical Engineering, Institute of Ergonomics, Manufacturing Systems and Automation, Otto-von-Guericke-University, 39106 Magdeburg, Germany (e-mail: firstname.lastname@ovgu.de)*

Abstract: To design a companion technology we focus on the appraisal theory model to predict emotions and determine the appropriate system behaviour to support Human-Computer-Interaction. Until now, the implementation of emotion processing was hindered by the fact that the theories needed originate from diverging research areas, hence divergent research techniques and result representations are present. Since this difficulty arises repeatedly in interdisciplinary research, we investigated the use of mathematical modelling as an unifying language to translate the coherence of appraisal theory. We found that the mathematical category theory supports the modelling of human emotions according to the appraisal theory model and hence assists the implementation.

Keywords: Human-Computer-Interaction, Mathematical Modelling, Appraisal Theory, Category Theory, Emotion prediction, User modelling

1. INTRODUCTION

The correlation between human cognition, emotion and behaviour has been in the focus of research for years (cf. Faghihi et al. (2011); Wehrle and Scherer (2001); Lazarus (1991)). Historically these subjects were believed to be separate, and research on these different fields led to a good, basic understanding of each. However, the challenge of Human-Computer-Interaction (HCI) is to understand the interdependence of these fields, to combine the available knowledge and to use this interdisciplinary understanding to design human-adaptable and human-supportive systems.

Research in the field of HCI is not only challenging due to the interdisciplinary aspects of the topic, but it is also hindered by the fact that human cognition, emotion and behaviour are investigated by researchers from different disciplines. This leads to dissimilar research methods, goals, specifications and results. While a psychologist's research on emotion will focus on the origin, identification and perception of emotions, engineers will want to understand if and how emotions can be recognised, predicted or influenced in terms of HCI. Both engineers and psychologists do fundamental research on the topic and their goals are interconnected but both will have difficulties to use the results obtained by the other group's investigations.

Mathematics as an unifying modelling language can assist to combine research results of groups originating from different disciplines. This may lead to a better understanding of the interdisciplinary field of study and enhance research on the topic. In this paper, we suggest that a mathematical formalisation of emotion theories can support the study of emotions regarding origin, alteration and interaction with cognition and behaviour. We further claim that a mathematical description of research results from the area of emotion studies will support and facilitate the study and development of affective systems.

Apart from this introductory section, this paper is organised in five further sections: The section "Designing affective systems" gives a brief introduction to the challenges and principles of designing systems that respond affectively to users. The "Appraisal Theory" section gives an introduction to the appraisal theory model as proposed by Scherer (2001). The "Category Theory" section gives an introduction to the mathematical category theory, its applications and benefits. The section on the "Categorical Appraisal Theory Model" gives an example of our postulated model, combining appraisal theory and category theory to find a model for the coherence of appraisal theory that is feasible in technical terms and that may be verified. In the final section conclusions regarding the suggested approach are drawn and the future work is briefly discussed.

⋆ This research was supported by the Transregional Collaborative Research Centre SFB/TRR 62 "Companion-Technology for Cognitive Technical Systems" funded by the German Research Foundation (DFG).

2. DESIGNING AFFECTIVE SYSTEMS

Research results prove that emotions have a significant influence on rational judgement, social interaction, perception, memory, creativity, learning and many other cognitive functions in humans, cf. Picard (1997). Hence, for computer systems to be recognised as affective, the system needs routines that provide information about the emotional processes involved in the decision making which the human counterpart does while interacting with the system. This is especially the case if the task for the computer system is not just to react emotionally to the user's input, but to be complaisant.

In affective computing, two main areas may be specified: 1) Detecting and responding to human emotions and 2) implementing emotional awareness in computer systems. While the latter is an object of intense discussions and progress is made rather slowly, the first has evolved during the last years (cf. Zeng et al. (2007); Fasel and Luettin (2003)). This can be seen in the fields of emotional speech recognition (cf. Ayadi et al. (2011); Schuller et al. (2009)), facial affect detection (cf. Busso et al. (2004)) and emotion detection from body gesture and physiological monitoring (cf. Walter et al. (2011)). However, each of these fields provides predictions regarding the expressed emotion based on different databases, empirical studies and research areas. The use of multi-modal sensor data hence implies that difficulties in the fusion process may arise and must be treated.

Using multi-modal sensor data for emotion recognition is beneficial because the manifestation of human emotions may be presented differently depending on the subject and situation. Some emotions may be identified easily within speech, while other emotions may be detected better through facial expression.

Apart from the different result representations that may arise due to the use of multi-modal data, another difficulty that arises is that it must be ensured that the emotion recognition is consistent and comprehensible. If not, the system will show inconsistent and inappropriate behaviour and may not be considered as behaving naturally and complaisant by its users. To avoid abstruse behaviour, strategies to resolve non-consistent emotion recognition of events through different modalities must be implemented. Depending on the modalities and sensors involved, this may be a challenging task.

One approach to obtain consistent system results is to rebuild the processing and evaluation of multi-modal data as it is carried out by humans when faced with multi-modal input. This also leads to results that humans will regard as "natural" and comprehensible.

There are several emotion theories, explaining the processing done by humans that lead to an emotional reaction. For our task to design a companion technology (cf. Wendemuth and Biundo (2011)) we focus on the appraisal theory model to predict emotions and choose the appropriate system behaviour to support the HCI (cf. Lazarus (1991); Scherer (2001); Roseman and Smith (2001)). The appraisal theory model allows both an understanding of when emotions arise and how emotions can be categorised, cf. Scherer (2005).

3. APPRAISAL THEORY

Emotions are a complex research topic which has been of interest to scientists from diverging disciplines for decades. During this time several theories were developed to explain the different aspects of the topic, such as the origin of emotions, the reason for emotions, what emotions are and how they are formed and expressed.

The theories developed were formulated for Human-Human Interaction, but several experiments showed that these theories are also applicable to HCI (cf. Cowie et al. (2001)).

Almost all cognitive emotion theories assume that the specific kind of emotion experienced depends on the result of an evaluation or appraisal of an event. Through this appraisal process the significance of the event for the survival and well-being of the organism is evaluated.

A subcategory of the cognitive emotion theories are known as appraisal theories. These represent the evaluation process of events as a cognitive, sequential, criteria based decision process. The theory attempts to specify the nature of the criteria used during the evaluation process in terms of appraisal variables/dimensions. Furthermore, it has been shown that appraisal theories are powerful generators for empirical research (cf. Scherer et al. (2001); Scherer (1987)).

One major criticism of the basic appraisal theories is that the models proposed are static models. The theories explain the evaluation process of one specific event, but lack methods to explain the dynamic processing of parallel events, the prioritisation of events and the different reaction types based on the previous evaluation step.

With the multilevel-sequential-checking model described in Scherer (2001) it can be explained in which sequence certain (emotional) expressions occur and why. This model is based on stimulus-evaluation checks (cf. Scherer (1988)) and the three level emotion-processing system proposed by Leventhal (1984). The three levels are the sensory-motory, the schematic and the conceptual level.

In Scherer (2001) it is described how the stimulus-evaluation checks may be processed on all three levels of the emotion-processing system. Furthermore it is derived that reactions which are based on the stimulus evaluation done at the sensory-motory level correspond to events which influence or are relevant to the basic needs of the subject. Events which are relevant for the acquired needs or motives are processed at the schematic level, while events that have relevance to the conscious goals or plans of the subject are evaluated at the conceptual level.

Hence, this rather technical model of the appraisal processing provides explanations for the different latencies of emotional reactions in humans and takes the history of appraisals and events into consideration. It also explains the prioritisation of events and allows parallel processing. Furthermore, the links between physiological respond patterns (later called features) and the evaluation process are established. These links are crucial for any implementation of the appraisal process in emotion detecting computer systems.

4. CATEGORY THEORY

Category theory has received much attention over the past decades and has developed rapidly within the last years. It has found applications in a variety of research fields, such as different areas of theoretical and applied computer science (cf. Walters (1992); Fiadeiro (2004)), neuroscience (cf. Brown and Porter (2003)), mathematics and logic (cf. Lane (1998); Lawvere and Schanuel (1997)) as well as philosophy (cf. Peruzzi (2006)).

Categories provide a convenient conceptual language (cf. Lane (1998)) and allow a formal description of the logic of paradigms, theories and methods. As a mathematical theory, the primary aim of category theory is not to provide formulas for calculation, but to explain and describe relationships. Due to this fact, category theory appears as a convenient tool for the investigation and formalisation of the coherence of appraisal theory.

Although the idea that mathematics itself consists of different branches (categories), has been known for centuries, it was not until 1945 when the explicit concept of categories was first introduced by Eilenberg and Mac Lane (cf. Lawvere and Schanuel (1997)). In Eilenberg and Mac Lane's paper "A general theory of natural equivalences" a tool was provided, that would allow the synthesis of mathematical branches and support the investigation of relationships. Category theory is therefore often considered as a general and conceptual language of mathematics.

Unlike other mathematical theories, category theory may not only be applied to mathematical objects, but rather allows an unified description of the coherence of theories and methods from almost any discipline. To get a deeper understanding of this far-reaching applicability, one must first understand what a category is.

One way of defining a category C is to explain what components a category is made of:

- A set of objects $o_1, ..., o_n \in O$ and
- a set of maps A (also called arrows), where each map $f \in A$ describes the structure-preserving relationship between two objects of O. In mathematical terms this means, that each $f \in A$ has one unique object as domain (*dom*) and one object as codomain (*cod*), notation: $f : o_1 \to o_2, f \in A, o_1, o_2 \in O$ and
- a composition-operator $\circ$ exists, such that for $f, g \in A$, if $cod(f) = dom(g)$, $g \circ f$ exists. In other words: If $f : o_1 \to o_2$ and $g : o_2 \to o_3$, then there exists $g \circ f : o_1 \to o_3$

The identity-axiom and the associativity of morphisms must be fulfilled:

- $\forall f \in A$: $f \circ id_{dom(f)} = id_{cod(f)} \circ f = f$ (identity)
- $f, g, h \in A : f \circ (g \circ h) = (f \circ g) \circ h$, if the composition is defined (associativity)

From these axioms it may be proved that exactly one identity-morphism for each object exists.

This gives a very basic introduction to categories. From the above one can already prove relationships within one category, such as the equivalence of objects or of maps. Furthermore, if the above is applied to an explicit example, one can discover the different types of morphisms by studying the properties of the relevant mappings.

Apart from investigating categories on a local basis, one may also discover relationships between categories. To do so, the term "functor" is introduced in category theory as a structure-preserving map between categories. One needs to distinguish between covariant and contravariant functors, where the latter reverses all morphisms of the category it is applied to. Furthermore for a functor $F : C \to D$, where C, D are categories, the following must hold:

- $\forall$ objects $o \in O$ of C an object $F(o)$ in D exists
- $\forall f \in A$, where A is the set of all maps in C, a F: $F(f) : f(o_1) \to f(o_2)$ exists and the following holds:
  - $\forall o \in C$ $F(id_o) = id_{F(o)}$
  - $\forall f : o_1 \to o_2, g : o_2 \to o_3 \in C$, $F(g \circ f) = F(g) \circ F(f)$

Please note that the above also implies that a functor preserves domain and codomain notions. This is even more explicit when using other approaches to define categories and functors.

From the above we are able to study categories on a local basis, namely investigate the relationships (arrows/maps) between objects of a category. Especially, we are able to derive the existence of a relationship and its type (type of morphism). Furthermore we are able to investigate the structure of categories in terms of similarities, differences and abnormalities in terms of objects and relationships (maps).

Using the construct of functors we are also able to investigate relationships between categories and may construct categories out of known categories, for example product categories or the dual category of a category, also known as the "opposite" category. The dual category of a category may be constructed by using the contravariant functor.

The above gives a basic understanding of the fundamental principles of category theory. There are (many) further concepts within the category theory and even more methods that may be useful for future investigations. The next section will show, that already with these rather principal methods of category theory, a formalisation of appraisal theory can be achieved.

5. CATEGORICAL APPRAISAL THEORY MODEL

To provide a system with information on emotions according to the appraisal model, the precise relation between user response patterns, appraisals and emotions must be known and feasible in technical terms, cf. Gratch and Marsella (2004).

Stating the appraisal theory in terms of category theory (cf. Awodey (2010)) leads to the description of emotions as mathematical categories, consisting of appraisals as objects, cf. figure 1. Appraisals in terms are derivable from the observable response patterns, cf. Scherer (2001). This leads to a formal, technical definition of an appraisal (object) as a vector of observables, as shown in figure 2.

Category theory additionally states that morphisms between the objects of a category exist. This condition is comprehensible, as the intuitive purpose of categories is

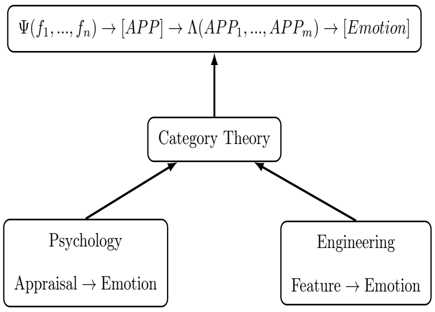

Fig. 1. Category theory as an unifying language for engineers and psychologists specifies a formalism for the coherence of the appraisal theory. Ψ is a function of features, f_i, determining an appraisal. The function, Λ, of appraisals, APP_j, defines an emotion.

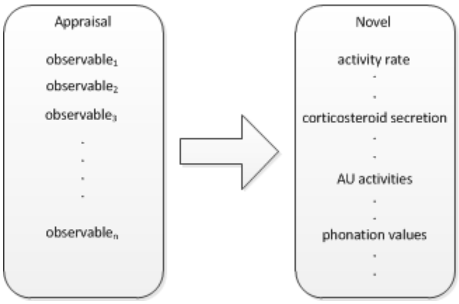

Fig. 2. Right: Appraisal vector defined by its observables. Left: Example of the appraisal vector for the appraisal "Novel" with some of the relevant observables as placeholders for the corresponding parameter values.

to assign equivalent subjects to common classes. The existence of morphisms between mathematical objects (appraisals) of a category (emotion) implies the existence of a structure-preserving transformation of one object (appraisal) to another. The change of the appraisal "novel" of an event to "non-novel" due to the passing of time is an example of this transformation, supposing that time is one of the observable features defining the appraisal.

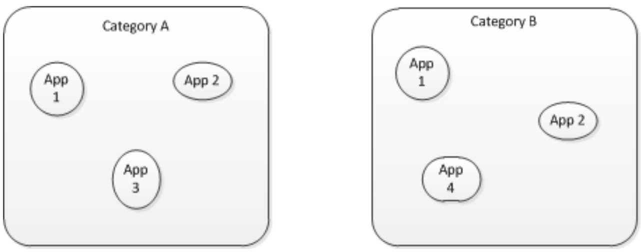

Fig. 3. Categories (emotions) defined by objects in terms of appraisal combination and value.

Knowing due to Scherer (2001), that emotions are definable in terms of a specific, observable response pattern in the human subject, one may combine this information with the knowledge about category theory and the given appraisal representation. This leads to a formal specification of emotions as combinations of appraisal vectors and of appraisal vectors as value-specific combinations of observables, as shown in figure 3.

Applying the methods of category theory, one may find the relationships between objects (appraisal vectors) of one category and the relationships between different categories (emotions), see figure 4. Especially the latter needs further attention and investigation, but significant results may be expected:

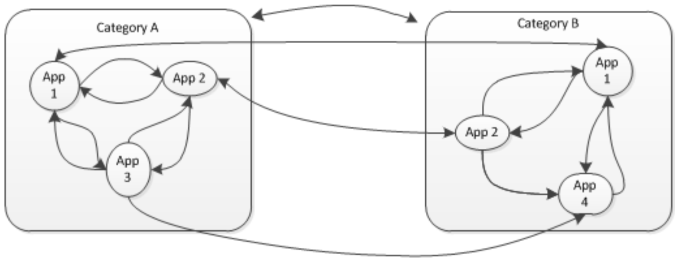

Fig. 4. Categories (emotions), objects (appraisals) and their relationships as arrows (morphisms)

The categories defined in terms of appraisal theory are mainly sub-categories of another. This means that they are sharing objects (appraisal vectors) and hence many of the local methods used to investigate category-specific relationships may be used to explore cross-category relationships. Explicitly this means that for two categories, for example anger and fear, certain appraisal values are allocated the same values, in this example (following Scherer (2001)) the appraisal values for goal hindrance and causation source.

The relationships found between these equivalent objects (appraisal vectors) in one category (anger), may also be assignable to the other category (fear). This allows to verify a certain similarity between the two emotions (categories). This similarity may be investigated further and will then allow a more sophisticated description of the type of similarity between emotions (categories) and the differences between the categories.

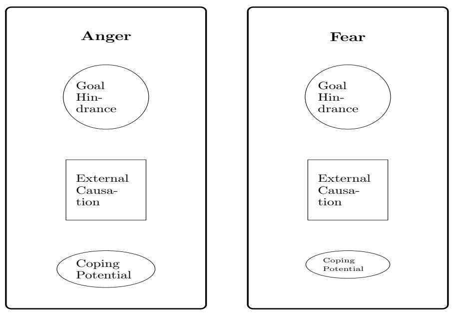

Fig. 5. Simplified visualisation of the emotions anger and fear with exemplified geometric representations of the appraisals coping potential, goal hindrance and causation source. The geometric shapes represent the manifestation of the appraisals. Similar shapes imply similar appraisal values, differences in the size of equivalent structures imply differences in parameter values.

Figure 5 shows a visualisation of the proposed model for a simplified example of the emotions (categories) anger and fear, based on Scherer (2001). The figure shows that the emotions anger and fear are defined by the appraisal vectors coping potential, goal hindrance and causation source and their manifestation. According to the previous statements based on Scherer (2001), the two latter appraisal values are equivalent in their manifestation, while the appraisal values for coping potential differ.

Due to the simplicity of this example, it can be observed directly that the emotions are equivalent, except for the manifestation of the appraisal for coping potential. This implies, that a change of the values for the coping potential appraisal (morphism of the object) will lead to a transformation from the emotion anger to the emotion fear, or vice-versa, depending on whether the parameter values increase or decrease. In this example, the described morphism of the object "coping potential" will be a simple function that adds or subtracts the relevant values of the observables.

The shapes used for the appraisals in the example are only for demonstration purposes. The real shapes of the appraisals are defined by the values of the observables and their manifestation. Similarities between appraisals are due to similarities in terms of the observable parameter values. An example of how an appraisal can be visualised using the parameter values of its observables is given in figure 6.

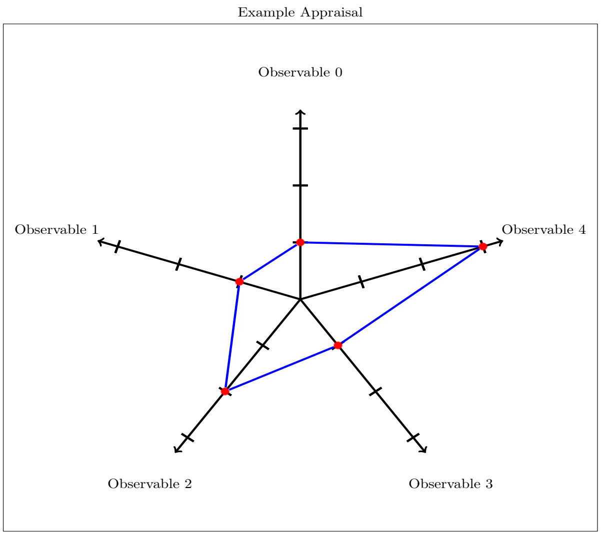

Fig. 6. Example of a geometric representation of an appraisal based on the values of the corresponding observables.

6. CONCLUSION

Although only done in principle, the above method of applying category theory to appraisal theory led to a technical specification of the coherence of appraisal theory and herewith improved the implementation of the appraisal theory.

Category theory especially assisted in developing the theoretical architecture for our new emotion processing and evaluation module and significantly reduced the time needed to extract the coherence of the psychological appraisal theory. This will be a benefit in future projects as well.

Furthermore, due to the categorical appraisal theory model, we are able to verify our derivations from the appraisal theory model in mathematical terms. It is also possible to check the model used for consistency, which enables us to guarantee consistent system states.

Additionally, the proposed model allows the visualisation of appraisals and herewith the visualisation of the relationships between appraisals in geometric terms. This is particularly useful for simulations regarding the alteration of appraisals. The results obtained will most certainly lead to a better understanding of the alteration of emotions, based on the appraisal theory assumptions.

Due to the geometric description of appraisals scientists are also provided with further mathematical investigation tools. For example, currently the similarities/differences of appraisals are given in terms of parameter value differences. With a geometric description of an appraisal and the use of the implicitly underlying analytic and geometric theories, it will be possible to define a metric, which in terms induces a measure. Herewith comparisons of appraisals can be stated in a more formal fashion.

Further analytic methods from the field of differential geometry (cf. Bär (2010); Michor (2008)) or differential topology (cf. Hirsch (1976)) may be applied to the geometric objects to study their characteristics. Differential geometry may be particularly useful if the investigated object has a significant local structure or property, while differential topology can be useful if the object should rather be investigated from a global point of view.

Apart from the need to further develop the proposed model and to seek for its limits, we are currently developing a two stage category model, aiming to reveal further details on how emotions are related to another and how emotions are connected to the event evaluation process in humans. The model will then also provide tools to prove these findings, provided the appraisal theory holds true.

REFERENCES

Awodey, S. (2010). *Category Theory (Oxford Logic Guides)*. Oxford University Press, second edition.

Ayadi, M.E., Kamel, M.S., and Karray, F. (2011). Survey on speech emotion recognition: Features, classification schemes, and databases. *Pattern Recognition*, 44(3), 572–587.

Bär, C. (2010). *Elementary Differential Geometry*. Cambridge University Press, first edition.

Brown, R. and Porter, T. (2003). Category theory and higher dimensional algebra: potential descriptive tools in neuroscience. In N. Singh (ed.), *Proceedings of the International Conference on Theoretical Neurobiology*, 80–92. National Brain Research Centre, Delhi.

Busso, C., Deng, Z., Yildirim, S., Bulut, M., Lee, C., Kazemzadeh, A., Lee, S., Neumann, U., and Narayanan, S. (2004). Analysis of emotion recognition using facial expressions, speech and multimodal information. In *Proceedings of the 6th international conference on Multimodal interfaces*, 205–211. ICIM'04, ACM, New York, NY, USA.

Cowie, R., Douglas-Cowie, E., Tsapatsoulis, N., Votsis, G., Kollias, S., Fellenz, W., and Taylor, J. (2001). Emotion recognition in human-computer interaction. *Signal Processing Magazine*, 18(1), 32–80.

Faghihi, U., Poirier, P., and Larue, O. (2011). Emotional cognitive architectures. In *Proceedings of the 4th in-*

ternational conference on Affective computing and intelligent interaction - Volume Part I, ACII'11, 487–496. Springer, Berlin, Heidelberg.

Fasel, B. and Luettin, J. (2003). Automatic facial expression analysis: a survey. *Pattern Recognition*, 36(1), 259–275.

Fiadeiro, J.L. (2004). *Categories for Software Engineering*. Springer, first edition.

Gratch, J. and Marsella, S. (2004). A domain-independent framework for modeling emotion. *Journal of Cognitive Systems Research*, 5, 269–306.

Hirsch, M.W. (1976). *Differential Topology*. Graduate Texts in Mathematics. Springer, New York, first edition. 6th printing (October 1997).

Lane, S.M. (1998). *Categories for the Working Mathematician*. Springer, second edition.

Lawvere, F.W. and Schanuel, S.H. (1997). *Conceptual Mathematics - A first introduction to categories*. Cambridge University Press.

Lazarus, R.S. (1991). *Emotion and adaption*. Oxford University Press New York.

Leventhal, H. (1984). A perceptual-motor theory of emotion. *Advances in Experimental Social Psychology*, 17, 117–182.

Michor, P.W. (2008). *Topics in Differential Geometry*. Graduate Studies in Mathematics. American Mathematical Society.

Peruzzi, A. (2006). The meaning of category theory for 21st century philosophy. *Axiomathes*, 16, 424–459.

Picard, R.W. (1997). *Affective Computing*. MIT Press.

Roseman, I.J. and Smith, C.A. (2001). Appraisal theory: Overview, assumptions, varieties, controversies. In A. Schorr and K.R. Scherer (eds.), *Appraisal processes in emotion: Theory, methods, research*, 221–232. Oxford University Press, London.

Scherer, K.R. (1987). Toward a dynamic theory of emotion: The component process model of affective states. *Geneva Studies in Emotion and Communication*, 1(1), 1–98.

Scherer, K.R. (1988). *Criteria for emotion-antecedent appraisal: A review*, 89–126. Kluwer, Dordrecht.

Scherer, K.R. (2001). Appraisal considered as a process of multilevel sequential checking. *Appraisal processes in emotion: Theory, methods, research*, 92–120.

Scherer, K.R. (2005). What are emotions? and how can they be measured? *Social Science Information*, 44(4), 695–729.

Scherer, K.R., Schorr, A., and Johnstone, T. (eds.) (2001). *Appraisal processes in emotions: Theory, methods, research*. Oxford University Press.

Schuller, B., Vlasenko, B., Eyben, F., Rigoll, G., and Wendemuth, A. (2009). Acoustic emotion recognition: A benchmark comparison of performances. In *Proceedings of the IEEE Automatic Speech Recognition and Understanding Workshop, ASRU 2009*, 552–557. Merano, Italy.

Walter, S., Scherer, S., Schels, M., Glodek, M., Hrabal, D., Schmidt, M., Böck, R., Limbrecht, K., Traue, H., and Schwenker, F. (2011). Multimodal emotion classification in naturalistic user behavior. In J. Jacko (ed.), *Human-Computer Interaction. Towards Mobile and Intelligent Interaction Environments*, volume 6763 of *Lecture Notes in Computer Science*, 603–611. Springer, Berlin.

Walters, R.F.C. (1992). *Categories and Computer Science*. Cambridge University Press.

Wehrle, T. and Scherer, K.R. (2001). Towards computational modeling of appraisal theories. In K.R. Scherer, A. Schorr, and T. Johnstone (eds.), *Appraisal processes in emotions: Theory, methods, research*, 350–365. Oxford University Press.

Wendemuth, A. and Biundo, S. (2011). A companion technology for cognitive technical systems. In *COST 2012 Conference "Cross Modal Analysis of Verbal and Nonverbal Communication"*. Dresden, Germany.

Zeng, Z., Pantic, M., Roisman, G.I., and Huang, T.S. (2007). A survey of affect recognition methods: audio, visual and spontaneous expressions. In *ICMI '07: Proceedings of the 9th international conference on Multimodal interfaces*, 126–133. ACM, New York, NY, USA.