Optimal Real-Time Control of Flexible Rack Feeders Using the Method of Integrodifferential Relations

Georgy Kostin* Harald Aschemann** Andreas Rauh** Vasily Saurin*

*Institute for Problems in Mechanics of the Russian Academy of Sciences, Moscow, Russia
(e-mail: {kostin, saurin}@ipmnet.ru).
**Chair of Mechatronics, University of Rostock, Rostock, Germany
(e-mail: {Harald.Aschemann, Andreas.Rauh}@uni-rostock.de)

Abstract: Rack feeders are of high practical importance as automated conveying systems. In this paper, control-oriented models are derived for an experimental setup representing the structure of a typical high bay rack feeder. On the basis of these models, feedforward control strategies are designed. The rack feeder is a viscoelastic structure consisting of two identical flexible beams which are attached to a horizontally movable carriage. The beams are rigidly connected at their tip by a pulley block which is necessary for the vertical positioning of a payload. To develop a real-time applicable control algorithm, a frequency analysis is performed for the original double-beam structure. As a consequence, a simplified Bernoulli beam model is derived with specific boundary conditions. The control objective under consideration is the positioning of the flexible beam structure at a desired position in such a way that the terminal mechanical energy stored in the beam is minimized. A modification of the Galerkin method which is based on an integrodifferential approach and a suitable finite element technique are employed to describe the viscoelastic structural vibrations and to design optimal control strategies. Results of numerical simulations are presented and compared with measured data.

Keywords: Rack feeder, control-oriented modeling, optimal control, viscoelastic structures.

1. INTRODUCTION

The design of control strategies for dynamic systems with distributed parameters has been actively studied in recent years. Processes such as oscillations, heat transfer, diffusion, and convection are part of a large variety of applications in science and engineering. The theoretical foundation for optimal control problems with linear partial differential equations (PDEs) and convex functionals was established by Lions (1971). Linear hyperbolic equations are treated, besides in Lions’ book, in Ahmed and Teo (1981), Butkovsky (1969). An introduction to the control of vibrations can be found in Krabs (1995). Oscillating elastic networks were investigated, e.g., in Lagnese et al. (1984).

Different approaches to the discretization of dynamical models with distributed parameters are developed to reduce the original initial-boundary value problem to a system of ordinary differential equations (ODEs). In this context, variational and projection methods are powerful tools to solve control problems for elastic structures. The method of integrodifferential relations (MIDR) was proposed in Kostin and Saurin (2006) for the design of optimal control laws for elastic beam motions. A projection approach was developed as a modification of the Galerkin method in the frame of the MIDR for dynamical systems described by linear parabolic PDEs in Rauh et al. (2010). In the current paper, this approach is combined with the finite element method developed in Saurin et al. (2011a, b). It is extended to modeling and optimal control of rack feeder systems with distributed elastic and inertial parameters which have already been considered by using an alternative system representation in Aschemann et al. (2011).

In Section 2, a typical structure of a flexible rack feeder system is considered. A Bernoulli beam model describing its structure is presented in Section 3. After that, a Fourier analysis is performed to reduce the beam model in Section 4 to the necessary degrees of freedom. The initial-boundary value problem describing the viscoelastic beam motion is formulated in Section 5. In the next section, an optimal control problem for damping of beam vibrations is given. A finite element algorithm is described in Section 7. The proposed control strategy is demonstrated, numerically verified, and experimentally validated in Section 8. Finally, the paper is concluded with an outlook on future research in Section 9.

2. A TYPICAL STRUCTURE OF FLEXIBLE RACK FEEDERS

The experimental setup which has been built up at the Chair of Mechatronics of the University of Rostock represents the structure of a typical high bay rack feeder as shown in Fig. 1 (left). The flexible structure consists of two identical beams clamped vertically to a horizontally movable carriage. The plane motions of the system are described in the noninertial Cartesian reference frame with the vertical axis $x$, the
The beams are rigidly connected at their tips by a pulley block which is necessary for the vertical positioning of a cage. The cage represents a payload sliding along the left horizontal axis z and the origin O located at the point where the beams are attached to the carriage.

The beams are clearly connected by a pulley block which is hinged to the other beam by means of a rigid rod. The following parameters of the structure are introduced: the length of the beams l, their cross section area A, the moment of area I, the distance 2b between the beams, Young’s modulus E and the volume density ρ of the beam material, the masses m_i and m_a of the pulley block and the cage, their moments of inertia I_i and I_a, the vertical position a(t) of the cage, and the height l/2 of the hinging.

The boundary conditions at the bottom and top of the beam have the form

\[ x = 0 : \quad w_j(0) = 0, \quad w'_j(0) = 0, \quad u_j(0) = 0; \]
\[ x = l : \quad w_j(l) = w_j(l), \quad 2bw'_j(l) = 2bw'_j(l) = u_j(l) - u_j(l), \]
\[ m_i \omega^2 (u_i(0) + u_i(l)) = 2f(l) + 2f(l), \]
\[ m_a \omega^2 (l) = -s_i(l) - s_i(l), \]
\[ J_i \omega^2 (l) = s_i(l) + s_i(l) + b(f(l) - f(l)). \]

In the proposed model, the lateral and longitudinal motions are further constrained by the interelement conditions

\[ x = a : \quad w_j(a) = w_j(a), \quad u_j(a) = u_j(a); \]
\[ w_j(a) = w_j(a), \quad w_j(a) = w_j(a), \]
\[ m_a \omega^2 (a) = s_i(a) + s_i(a) - s_i(a) - s_i(a), \]
\[ m_a \omega^2 (a) = f(a) - f(a), \quad f(a) = f(a), \]
\[ J_i \omega^2 (a) = s(a) - s(a), \quad s(a) = s(a). \]

The terms with the coefficient m_a appear in (3) due to the rigid coupling of the first beam with the cage. The coefficients \( c_{j\beta}^{(\alpha)} \) and \( d_{\beta} \) in (2) and the eigenfrequency \( \omega \) are defined form the degeneracy condition for the system of boundary and interelement constraints (3), (4).

4. REDUCED DOUBLE BEAM MODEL

To derive a control-oriented model for the experimental setup, let us estimate the three lowest eigenmodes of the rack feeder structure which are only excited noticeably in typical motions. The following geometrical and mechanical parameters provided by measurements are used in the vibration analysis:

\[ l = 1.07 \text{ m}, \quad b = 0.0245 \text{ m}, \quad \rho = 2700 \text{ kg} \cdot \text{m}^{-3}, \]
\[ m_i = 0.906 \text{ kg}, \quad m_a = 0.95 \text{ kg}, \quad E = 70 \text{ GPa}, \]
\[ A = 3.10 \times 10^{-4} \text{ m}^2, \quad I = 2.138 \times 10^{-9} \text{ m}^4. \]

It can be shown numerically that the real moments of inertia \( I_i \) and \( I_a \) do not sufficiently contribute to the values of the first three eigenfrequencies \( \omega_n \), \( n = 1, 2, 3 \). Therefore, they are set to zero in (3), (4).

The eigenvalues for the system (1), (3), (4) obtained by the Fourier method (see Courant and Hilbert (1989)) are summarised in Table 1. It can be seen from the table that the
longitudinal wave numbers $\mu_n$ are much smaller than the corresponding lateral ones $\lambda_j$ for $n \leq 3$. This means that the trigonometric functions of vertical extension and compression $u_j(x)$, $j = 1, 2, \ldots$ from (2) can be replaced rather accurately by linear approximations with respect to the coordinate $x$. Moreover, the lateral displacements $w_1(x)$ and $w_2(x)$ do not differ noticeably from each other for these modes. All this allows us to propose a reduced model of the rack feeder (see Fig. 1, right) as a single Bernoulli beam with the doubled geometrical parameters of the cross section area $2A$ and the moment of area $2I$.

### Table 1. Eigenvalues for elastic and viscoelastic models

<table>
<thead>
<tr>
<th>Mode</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two coupled elastic beams (Fourier method), Sec. 3, 4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda$ in m$^{-1}$</td>
<td>1.72</td>
<td>3.87</td>
<td>7.44</td>
</tr>
<tr>
<td>$\mu$ in m$^{-1}$</td>
<td>7.72\times10^3</td>
<td>3.91\times10^2</td>
<td>0.145</td>
</tr>
<tr>
<td>$\omega$ in s$^{-1}$</td>
<td>39.30</td>
<td>198.98</td>
<td>737.94</td>
</tr>
<tr>
<td>One double elastic beam (Fourier method), Sec. 4</td>
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<tr>
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<tr>
<td>$\omega$ in s$^{-1}$</td>
<td>39.30</td>
<td>198.98</td>
<td>737.94</td>
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<tr>
<td>Viscoelastic beam model (Fourier method), Sec. 5</td>
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<td></td>
</tr>
<tr>
<td>$\lambda$ in m$^{-1}$</td>
<td>39.30</td>
<td>198.98</td>
<td>737.94</td>
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<tr>
<td>$\mu$ in m$^{-1}$</td>
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<tr>
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<td>Viscoelastic beam model (FEM), Sec. 7, 8</td>
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<tr>
<td>$\lambda$ in m$^{-1}$</td>
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<td>0.713</td>
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</tr>
</tbody>
</table>

The equations of motion for this new beam model are

$$s^2 - 2Aw\omega^2 w = 0, \quad 2Ew^\prime - s = 0, \quad x \in (0, a) \cup (a, l),$$

(6)

where $w(x)$ is the lateral displacement of the introduced beam and $s(x)$ is its bending moment. The simplified boundary and interelement conditions have the form

$$x = 0: \quad w(0) = 0, \quad w'(0) = 0;$$

$$x = l: \quad m_0 w(l) = -s'(l), \quad s(t, l) = -\kappa w'(l);$$

$$x = a: \quad w(t, a - 0) = w(t, a + 0), \quad w'(t, a - 0) = w'(t, a + 0),$$

$$s(t, a - 0) = s(t, a - 0), \quad m_0 w'(a) = s'(a - 0) - s'(a - 0).$$

Here $\kappa$ is the effective elastic coefficient of the hinged support at the upper beam end. Its value $\kappa = 2AEb^2/l$, directly follows from the quasistatic approximation of the longitudinal force $f_j$ in (3).

As it is shown in Table 1, both the original and the reduced models have rather close values of the wave numbers $\lambda$ and the eigenfrequencies $\omega$ for the first three modes. Furthermore, the reduced model includes only one unknown displacement function $w$ as compared with four in the original structure. As a consequence, this simplified model can be applied more efficiently to control design without relevant loss of accuracy.

### 5. VISCOELASTIC MOTIONS

The flexible beam elements of the experimental setup are made of aluminium alloy that behaves in the design area of the working parameters more or less like a viscoelastic medium. As it is shown in Fig. 2, the experimental measurements confirm the exponential characteristic of amplitude damping during free vibrations of the structure. Let us consider the Kelvin–Voigt model of viscoelasticity to take into account this damping. Then, the dynamical system of PDEs describing the motion of the reduced beam structure proposed in the previous section can be represented by

$$\frac{\partial p}{\partial t} + \frac{\partial^2 s}{\partial x^2} = 0, \quad \eta := \frac{\partial w}{\partial t} + \nu(t) - \frac{p}{2A} = 0, \quad t \in (0, T),$$

$$\zeta := \frac{\partial^2 w}{\partial x^2} + \frac{\delta}{EI} \frac{\partial^2 c}{\partial x^2} - s = 2EI, \quad x \in (0, a) \cup (a, l), \quad (8)$$

where $w(t, x)$ is the displacement, $s(x)$ is the bending moment, $p(t, x)$ is the linear density of lateral momentum, and $\delta$ is the viscosity factor. Auxiliary functions $\eta$ and $\zeta$ are introduced here to shorten the subsequent notations. Analogously to the relations (7), the boundary and interelement conditions are stated as

$$\left[ m_i \left( \frac{\partial^2 w}{\partial t^2} + \frac{d\nu}{dt} \right) - \frac{\partial s}{\partial x} \right]_{x = a, l} = 0,$$

$$w \left|_{x = a, l} = w \left|_{x = a, l} = \frac{\partial w}{\partial x} \right|_{x = a, l} = \frac{\partial w}{\partial x} \right|_{x = a, l} = 0, \quad (9)$$

For the system (8), (9), the free lateral displacement $w$ can be given in the form

$$w = \left( c_1 e^{\lambda_i x} + c_2 e^{-\lambda_i x} + c_3 e^{\lambda_i x} + c_4 e^{-\lambda_i x} \right) e^{\nu t},$$

$$\lambda = \sqrt{\frac{2}{EI} \left( \frac{E + \delta} {\omega} \right) \sqrt{-A \rho (E + \delta) \omega}}, \quad \zeta = -\nu + i\omega, \quad (10)$$

where $\omega$ is the frequency and $\nu \geq 0$ is the damping coefficient of the free beam motion. The complex constants $c_k\, (k = 1, 2, \ldots, j = 1, 2, 3, 4)$, can be found from the conditions (9). It can be proven that for any $n = 1, 2, \ldots, n$ the eigenvalues $\lambda_n$ are neither purely imaginary nor real and that $\nu_n > 0$ unless $\delta_n = 0$. In the viscoelastic model under study, only a finite number of the lowest eigenmodes ($n \leq n'$) are damped oscillations with $\omega_n \neq 0$, the other natural modes ($n > n'$) are monotone transients with $\omega_n = 0$. 
Experimental data of the free vibrations of the rack feeder structure allows for estimating the first eigenfrequency and the damping factor as \( \omega_1 \approx 39.3 \, \text{s}^{-1} \) and \( \nu_1 \approx 0.7 \, \text{s}^{-1} \), respectively. Under consideration of the model parameters given in Section 4, the best estimate for the viscosity factor corresponding to these values is \( \delta_1 = 0.14 \, \text{N} \cdot \text{m}^{-1} \cdot \text{s} \). The first three frequencies \( \omega_j \) and damping factors \( \nu_j \) obtained by the Fourier method for the viscoelastic beam model are also given in Table 1. These eigenfrequencies are strictly below the respective values for the elastic double beam model. Note that only the five lowest modes are oscillatory \( (n' = 5) \) for this viscosity factor.

Fig. 2. Free vibrations of the beam structure: experimental result.

6. OPTIMAL CONTROL PROBLEM

To formulate an optimal control problem in closed form, it is necessary to specify the initial conditions in an appropriate form

\[
w(0, x) = w^0(x), \quad p(0, x) = p^0(x), \quad (11)
\]

where \( w^0 \) is the lateral beam displacement at the initial time \( t = 0 \) and \( p^0 \) is the initial momentum distribution.

Consider controlled motions of the viscoelastic beam described by the PDE system (8) with boundary and initial conditions (9), (12). In this system, the function \( v \), corresponding to the velocity of the carriage, is the control input and \( T \) is the terminal instant of the control process. The problem is to find an optimal control \( \nu^*(t) \) in a given control set \( V \) that moves the carriage from its initial state defined in (11) to the terminal position \( z_T \) with vanishing velocity \( \nu(T) = 0 \) in the fixed time \( T \) and minimizes an objective function \( J_0[\nu] \) (terminal mechanical energy of the whole structure):

\[
J_0[\nu] \rightarrow \min_{\nu \in V}, \quad J_0 = W(T),
\]

\[
W = \int_0^T \left( \nu \delta t + \frac{m}{2} \left( \frac{\partial w}{\partial t} \right)^2_{x=x_0} + \frac{m}{2} \left( \frac{\partial w}{\partial t} \right)^2_{x=x_l} + \frac{K}{2} \left( \frac{\partial w}{\partial x} \right)^2 \right) dt. \quad (12)
\]

The total energy \( W \) contains not only a term defined by the distributed parameter (the density of beam energy)

\[
y' = EI \left( \frac{\partial^2 w}{\partial x^2} \right)^2 + \frac{p^2}{4A\rho} \quad (13)
\]

but also the terms specified by the lumped system components such as the inertia of the cage and the pulley block as well as the elasticity of the hinged support at the upper beam end (see Fig. 1).

To solve the initial-boundary value problem, we apply the MIDR in which the local equalities \( \eta = \xi = 0 \) and initial conditions (12) are replaced by integral relations, whereas the first relation in (8), representing the equilibrium equation, as well as the boundary conditions (9) are satisfied exactly.

7. FINITE-ELEMENT SEMI-DISCRETIZATION (FEM)

Let us eliminate the function \( s \) by taking into account the first equation in (8) and the boundary conditions at \( x = l \) as follows

\[
s = - \int_0^1 \int_0 \left[ h(t, x) \right]_{x=0} dy \delta v_i - K \frac{\partial w}{\partial x} \left|_{x=l} \right.,
\]

\[
h = \frac{\partial p}{\partial t} + m \left( \frac{\partial w}{\partial t}^2 + \frac{d^2 v}{dt^2} \right) \delta (x-a). \quad (14)
\]

Afterwards, define a space mesh with the nodes \( x_0 = 0 \), \( x_m = l \), \( 0 \leq x_j < x_{j+1} \), \( J = (x_{j-1}, x_j), \ j = 1, \ldots, M \), where \( M \) is the number of elements. To find an approximate solution of the optimization problem (12) on the basis of the finite element discretization by Saurin et al. (2011), the functions \( w \) and \( p \) are approximated by splines

\[
w \in S_{W}^{(N+2)} = \left\{ w(t, x) : w = w_0(x) + \sum_{j=1}^{N+2} w_j (t) (x/l)^j, \right\}, \quad w \in C^1, x \in [0, l], w_0(t) = w_1(t) = 0
\]

\[
p \in S_p^{(N)} = \left\{ p(t, x) : p = p_0(x) + \sum_{j=1}^{N} p_j (t) (x/l)^j, \right\}, \quad x \in I_j, i = 1, \ldots, M
\]

\[
w(t) = \{w_1, \ldots, w_N\}, \quad w_i = \{w_{2i}, \ldots, w_{2i+N}\}
\]

\[
p(t) = \{p_1, \ldots, p_0\}, \quad p_i = \{p_{2i}, \ldots, p_{2i+N}\}, \quad i = 1, \ldots, M
\]

where \( w(t) \), \( p(t) \) are vector functions defining the unknown displacement \( w(x, w) \) and the linear momentum density \( p(x, p)(t) \) with the order \( N \) of the polynomial approximation on each element.

A projection approach is used to reduce the original PDE system to a system of ODEs with initial conditions in the form

\[
\int_0^1 \eta(t, x, w, p, \nu) \chi(x) dx = 0, \quad \int_0^1 \xi(t, x, w, p) \chi(x) dx = 0,
\]

\[
\forall \chi \in S_{\chi}^{(N)} = \left\{ \chi(x) : \chi = \sum_{j=1}^{N} \chi_j x^j / l, x \in I_j, i = 1, \ldots, M \right\}.
\]

(16)
Here, $\eta$ and $\xi$ are obtained by substituting the relations (14) and (15) into (8). According to (15), homogeneous initial conditions are imposed on the displacement and momentum vector functions

$$w(0) = 0 \quad \text{and} \quad p(0) = 0. \quad (17)$$

The following relative integral error is proposed to estimate the quality of the approximation to the PDE system (8)

$$\Delta = \frac{\Phi}{\Psi}, \quad \Phi = \int_{0}^{T} \phi(t,x)dxdt,$$

$$\Psi = \int_{0}^{T} W(t)dt, \quad \phi = A\rho \eta^2 + EI \xi^2. \quad (18)$$

To design the optimal control law, a consistent polynomial control input is considered according to

$$v(t) = T - \sum_{k=1}^{K} v_k t^k, \quad \text{s.t.} \quad v(T) = 0. \quad (19)$$

The control $v$ contains only $K$ independent coefficients $v_k$ since one parameter, for example, $v_{K+1}$ can be expressed explicitly via the others from the terminal condition

$$\int_{0}^{T} v(t,v_{K+1}) dt = w_T. \quad \text{After solving the initial value problem (16), (17) for the undefined vector } v = \{v_1,\ldots,v_k\} \text{ of control parameters, the resulting vector functions } \hat{w}(t,v) \quad \text{and} \quad \hat{p}(t,v) \quad \text{are used to minimize a modified objective function}$$

$$J_1(v) = \min \frac{\gamma_1}{T} J_0(v) + \frac{\gamma_2}{T} \Psi(v) + \frac{\gamma_3}{T} \Phi(v), \quad (20)$$

where the last terms with the dimensionless weighting coefficients $\gamma_{1,2}$ are added to influence the numerical condition number of the optimal solution. The character ‘tilde’ in (20) means that the corresponding functionals $\hat{J}_0$, $\hat{\Psi}$, and $\hat{\Phi}$ are expressed via the solutions $\hat{w}$, $\hat{p}$ after substituting them for the unknown values in (15). The resulting optimal value $v^*$ of the control vector describes the optimal control function $v^*(t) = v(t,v^*)$ in the following numerical simulation and the corresponding experiments.

8. NUMERICAL AND EXPERIMENTAL RESULTS

To verify the quality of the optimal feedforward control strategy described above, numerical simulations are performed. The solutions are obtained analytically from the set of ODEs (16) using symbolic formula manipulation. It is considered that the beam is undeformed and motionless at the initial instant $t = 0$, i.e., $w_0(x) = p_0(x) = 0$ holds. The following parameters of the control problem (12) and the approximations (15) are given: $T = 2 \, \text{s}$, $z_2 = 0.5 \, \text{m}$, $K = 6$, $\gamma_1 = 0.01$, $\gamma_2 = 100$, $M = N = 4$. The accuracy of the first three eigenfrequencies $\omega_n$ and the damping coefficients $\nu_n$ obtained by the FEM discretization are shown in Table 1. The resulting objective energy function is $J_0 \approx 4.18 \times 10^{-4} \, \text{J}$ for the optimal control $v^*(t)$ shown in Fig. 3. The average energy error $\Phi/T \approx 2.39 \times 10^{-13} \, \text{J}$ is sufficiently small in comparison with the average mechanical energy of the system $\Psi/T \approx 0.0794 \, \text{J}$. For the given approximation parameters $M$ and $N$, the relative error of the numerical solution is equal to $\Delta = 3.02 \times 10^{-6} \%$. The displacement of the free end is shown in Fig. 4.

![Optimal control: carriage velocity $v^*(t)$](image1)

![FEM simulation: viscoelastic displacement $w(t,l)$ of the pulley.](image2)

The residual vibrations after the control process ($t > T$) are rather small with respect to the maximal quasistatic deflection of the beam end during the carriage motion as it can be seen from Fig. 4. In Fig. 5, the time history of the bending moment $s$ at the output point $x = x_j = 0.09 \, \text{m}$ is presented. The optimal control $v^*$ shown in Fig. 3 has been used as the control input function in the experimental setup. The measured data taken from a strain gauge situated at the position $x = x_j$ during the experiment with the same control $v^*$ are presented in Fig. 6. It is important to note that the strain gauge signal is proportional to the bending moment in the beam cross section at the point $x_j$. It can be seen that the bending moment output obtained in the numerical simulation (Fig. 5) is very similar to the experimental signal (Fig. 6).
However, noticeable higher oscillation frequencies are also excited in the experiment. Nevertheless, the residual vibrations of the rack feeder are not so essential, and the system reaches the desired position \( z_f = 0.5 \, \text{m} \) with high accuracy in the defined time \( t \leq T \), as shown in Fig. 7.

9. CONCLUSIONS AND OUTLOOK

In this paper, a Fourier technique has been applied to derive a control-oriented model of a typical high bay rack feeder. A modeling and optimization algorithm has been developed for feedforward control of distributed viscoelastic structures based on the MIDR, a projection approach, and a novel finite element technique. A numerical verification and experimental validation on a test rig has been performed for the proposed control strategy. In future work, an offline identification of system parameters from measured data and the design of a robust observer-based feedback control structure are planned on the basis of the MIDR.

ACKNOWLEDGEMENTS

This work was supported by the Russian Foundation for Basic Research, project nos. 09-01-00582, 10-01-00409, 11-01-00472, 11-01-08117, the Leading Scientific Schools Grants NSh-3288.2010.1, NSh-64817.2010.1, as well as by the Alexander von Humboldt Foundation.

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