Fault Indicators and Adaptive Thresholds from Hybrid System Models

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Abstract: Initially, incremental bond graphs were introduced to support frequency domain sensitivity analysis of linearised time-invariant models. Subsequent publications have shown that they can be used for other purposes as well such as the determination of parameter sensitivities of analytical redundancy relations (ARRs) in symbolic form.

This paper extends previous work to fault detection and isolation (FDI) from hybrid system models. In order to ensure robust FDI in the presence of parameter uncertainties and to avoid false alarms of faults reported to a diagnosis system, properly chosen thresholds for the time evolution of residuals of ARRs are needed. Hybrid systems exhibit several operation modes. Distinctive continuous time behaviour in different operation modes require adapted thresholds for fault indicators. The paper shows how an incremental bond graph can be used to set up thresholds for residuals that account for parameter uncertainties as well as for discrete mode changes. The approach is illustrated by means of an example easy to survey.

Keywords: Hybrid systems, FDI, ARRs, adaptive thresholds, incremental bond graphs.

1. INTRODUCTION

Bond graph model-based quantitative fault diagnosis has been subject of ongoing research. Especially the progress achieved during recent years by work at Indian Institute of Technology, Kharakpur, India and at University of Lille and Ecole Centrale de Lille, France has to be mentioned. See Samantaray and Ould Bouamama (2008); Ghoshal (2006); Samantaray et al. (2006). For the derivation of Analytical Redundancy Relations (ARRs), Samantaray introduced what he termed Diagnostic Bond Graphs (DBGs) with sensors in reversed and storage elements in preferred derivative causality. Recently, Djezeri and his co-workers presented an approach to robust fault detection and isolation (FDI) in the presence of parameter uncertainties that is based on a specific bond graph representation and enables to set up a state space model in the standard interconnection form, also known as linear fractional transformation (LFT) form (Chapter 3 of Borutzky (2011b)). The advantage of bond graphs in LFT form is that ARRs derived from the bond graph consist of a nominal part and a part that accounts for parameter uncertainties and can be used to find adaptive ARR thresholds.

As bond graph modelling considers the exchange of energy between system components, publications reported in the literature have mainly focused on FDI in systems represented by continuous time models. Back in 2002, Nasrasimhan used hybrid bond graphs, an extension of bond graphs with junctions switched on and off by local automata, for model-based diagnosis of hybrid systems (Narasimhan (2002)). Since disconnection and reconnection of bond graph parts by controlled junctions may require a partial reassignment of computational causalities and thus affect the formulation of ARRs, Low et. al. recently proposed a modification of the Standard Causality Assignment Procedure (SCAP) that assigns storage elements derivative and junctions preferred causalities aiming at minimising the propagation of computational causality changes (C.B. Low and D. Wang and S. Arogeti and J.B. Zhang (2010)). In Chapter 7 of Borutzky (2011b), Ghoshal and Samantaray apply the LFT bond graph approach to robust FDI proposed by Djeziri et. al. to a hybrid two tank system with a drain from tank 1 to tank 2 that becomes effective when the fluid level control of tank 1 fails.

As an alternative to the use of hybrid bond graphs and a specific SCAP adapted for FDI, the author recently proposed to derive ARRs for hybrid systems from bond graphs in which switching devices are represented by a Boolean controlled transformer and a resistor in fixed conductance causality (Borutzky (2011a)). The latter representation was initially introduced by Ducreux et. al. (Ducreux et al. (1993)) and has been known for a long time. The advantage of the proposed approach to FDI for hybrid systems is that static causality can be assigned by the unmodified SCAP and that ARRs can be derived that hold for all system operation modes.

Incremental bond graphs were initially introduced by the author for the purpose of frequency domain sensitivity analysis of linearised time-invariant models (Borutzky and Granda (2002)). Further research has shown that they are suited for various other purposes as well, e.g., for the determination of parameter sensitivities of ARRs in symbolic form (cf. Chapter 4 of Borutzky (2011b)).

This paper extends the incremental bond graph approach to hybrid systems and shows that they can also be used to set up adaptive thresholds for ARR residuals that account for parameter uncertainties as well as for discrete mode changes. This is an issue of practical relevance because on the one hand, evaluation of ARRs should be robust
with regard to parameter uncertainties, i.e. false indication of faults should be avoided. On the other hand, true faults must be detected. Moreover, hybrid systems operate in different modes and their continuous time behaviour in different modes may be quite distinctive. Therefore, different ARR thresholds adapted to different modes may be required.

The paper is organised in the following manner. The next section briefly recalls some features of incremental bond graphs and addresses the issue of how switching devices and parameter faults considered as discrete events can be modelled. The main part of the paper illustrates the approach by means of an example. The nominal part of ARRs is derived from a bond graph with nominal parameters associated with the incremental bond graph. ARR thresholds are obtained after summing up flows or efforts respectively at junctions of the incremental bond graph. The paper concludes with a brief summary of the advantages of the approach.

2. INCREMENTAL BOND GRAPHS

In this brief recall of incremental bond graph basics, it is assumed that energy sources, energy stores and resistors are linear 1-port elements for the sake of simplicity. As to transformers and gyrators, elements with two ports are considered.

The starting point is that a somehow caused parameter variation $\Delta \Theta$ entails both power variables at the ports of an element to be perturbed due to its interaction with other elements in the model. That is, the value $v(t)$ of a power variable, being either an effort $e$ or a flow $f$, deviates from its nominal value $v_n(t)$ in normal operation mode by a variation $\Delta v(t)$.

$$v(t) = v_n(t) + \Delta v(t)$$

(1)

Incremental bond graphs represent energy flows $(\Delta e(t)) \times (\Delta f(t))$. They are obtained by replacing elements in an original bond graph by their incremental model. The incremental model of 1- and of 0-junctions remains a 1-junction, and 0-junction respectively as these elements do not depend on parameters. Likewise, sources that do not depend on a parameter become sources of value zero. The incremental model of other bond graph elements differ from the element by modulated sinks added to a junction. These sinks account for the parameter variation. They are modulated by a power variable of the original bond graph. As a result, an incremental bond graph has the same structure as the original bond graph. It only has got additional modulated sinks.

2.1 Incremental Models for Bond Graph Elements

The development of an incremental model for bond graph elements is illustrated by considering a linear 1-port $\mathcal{C}$ element. Let $C_n$ be its nominal capacitance. Given that a variation $\Delta C$ happens then the constitutive equation in derivative causality takes the form

$$f_C = f_{C_n} + \Delta f_C = (C_n + \Delta C)(\dot{e}_{C_n} + \Delta \dot{e}_C).$$

(2)

Reformulation gives

$$\frac{\Delta e_C}{\Delta f_C} = \frac{C_n}{0} \quad \mathcal{C} : C_n$$

$$\mathcal{M}_{Se : \delta C \times f_C}$$

$$\mathcal{M}_{Se : (\Delta m)_{e2}} \quad \mathcal{M}_{Se : (\Delta m)f_1}$$

Fig. 1. Incremental bond graph model of a linear 1-port $\mathcal{C}$ element

$$\frac{\Delta e_1}{\Delta f_1} = \frac{m_n}{\mathcal{T}F} \quad 0 \quad \frac{\Delta e_2}{\Delta f_2}$$

$$\mathcal{M}_{Se : (\Delta m)e_2} \quad \mathcal{M}_{Se : (\Delta m)f_1}$$

Fig. 2. Incremental bond graph model of a 2-port transformer

$$\Delta f_C = C_n \times \dot{e}_C + \frac{\Delta C}{C} \times f_C = : \delta_C$$

where $C = C_n + \Delta C$. Equation 3 may be represented by the bond graph in Fig. 1.

Note, in FDI, energy storage elements are usually assigned preferred derivative causality in order to get rid of initial conditions that are not known or difficult to obtain in real time FDI. That is, computation of a bond graph model for FDI takes place in real time simultaneously with the operation of a real process that provides measured quantities as inputs into the model (cf. Section 7.5 in Borutzky (2011b)).

Likewise, the incremental model of a 2-port transformer may be obtained. Fig. 2 depicts the result where $\delta_m := \Delta m/(m_n + \Delta m)$.

2.2 An Incremental Model for Switches

Various approaches have been reported in the literature to extend bond graph modelling so that the abstraction of instantaneous changes between system modes is also supported. One approach known for a long time has been to represent switching devices by means of a modulated transformer controlled by a Boolean variable in conjunction with a resistor in fixed conductance causality accounting for the small ON-resistance of a switch (Garcia-Gomez, 1997; Ducrueux et al., 1993; Dauphin-Tanguy and Rombaut, 1993). The aim of these so-called hybrid system models encompassing continuous time behaviour and discrete events has been mainly the simulation of the dynamic behaviour. Recently, the author proposed to use the above representation of conceptual switches in bond graphs for FDI in hybrid systems (Borutzky (2011a)). This paper extends the approach towards the determination of adaptive thresholds for ARRs residuals that are robust in the presence of parameter variations.

In ON-mode, the switch model simply reduces to a resistor with the small nominal ON-resistance $R_{on}$. The constitu-
3. Incremental Bond Graphs for FDI

In previous work, incremental bond graphs have been used to determine first order frequency domain parameter sensitivities of output functions, transfer functions (Borutzky and Granda (2002)) or of ARR residuals (Borutzky (2009), Chapter 4 of Borutzky (2011b)). In that context, $\Delta \Theta$ is considered small parameter variations or parameter uncertainties and incremental models of bond graph elements may be obtained by taking the total differential of their constitutive equation. Then, for the incremental of each output variable, in particular for the incremental $\Delta r$ of an ARR residual $r$, an equation can be derived from the incremental bond graph. As parameter variations are inputs delivered to the incremental bond graph by sources that are modulated by a power variable of the original bond graph, the incremental of an ARR residual can be expressed as a weighted sum of parameter variations. The output parameter sensitivities are just the weights (cf. Chapter 4 of Borutzky (2011b)).

In the context of FDI in hybrid system models, parameter variations may not be just small deviations from nominal values but may account for any kind of parameter fault that can even significantly affect the system’s behaviour. Moreover, as coefficients of parameter variations may include moduli $m_i \in \{0, 1\}$ of transformers representing switches, variations of ARR residuals are system mode dependent.

As parameter variations are not confined to small uncertainties, incremental models of bond graph elements can not be obtained by just taking the total differential of their constitutive equation. If the constitutive equation of a bond graph element, however, is nonlinear then the variation of its output power variable cannot be simply expressed as a weighted sum of variations and be presented by appropriate bond graph elements attached to a 0- or a 1-junction.

![Fig. 4. Extended incremental bond graph model of a switch accounting for a non-zero flow in OFF-mode](image)

### 3.1 Extension of the Switch Model

The incremental model of a switch given in the previous section so far only accounts for a fault in the ON-resistance. In addition, in OFF-mode it may happen that the flow does not completely vanishes. By consequence, model parts remain connected. Then, in OFF-mode, the switch can be considered a resistor with a very high OFF resistance $R_{\text{off}}$. For this resistor, an incremental model can developed in the same way as for the resistor $R : R_{\text{on}}$. In the resulting model, the resistor $R : R_{\text{off}}$ can be neglected as the nominal value of $R_{\text{off}}$ is very high. However, $\delta R_{\text{off}} := \Delta R_{\text{off}} / R_{\text{off}}$ may take a value leading to a flow in OFF-mode that the modeller may not want to neglect. The extended incremental model of switch depicted in Fig. 4 accounts for a possible non-zero flow in OFF-mode.

### 3.2 Structural faults

Parameter faults do not change the model structure. It is just the value of a device parameter that changes for some reason. Possible faults that affect the model structure, e.g. a sudden occurring leakage in a device may be captured by means of a modulated transformer controlled by a Boolean variable that switches the effect on or off. If it is a leakage, for instance, its amount is determined by a resistor. If the resistance of the latter remains time-invariant then the incremental model of this effect is just a Boolean controlled transformer connect to a resistor.

4. Adaptive Thresholds for ARRs Residuals

This section shows by means of an example how an incremental bond graph can be used for the determination of adaptive thresholds for ARRs derived from a hybrid system model that are robust with regard to parameter uncertainties. As an example, a boost converter with pairwise switching devices commonly used in power electronics is considered. Fig. 5 depicts its circuit schematic, Fig. 6 displays a bond graph model with preferred integral causalities and Fig. 7 shows a Diagnostic Bond Graph (DBG) with preferred derivative causality and sensors with inverted causality for the inductor current $i_L$ and the voltage drop $u_C$ across the load capacitor. The two switching elements have been modelled by means of a modulated transformer and an ON-resistor $R : R_{\text{on}}$ in fixed conductance causality. The current through these elements is assumed to be zero in OFF-mode. The auxiliary capac-

![Fig. 3. Incremental bond graph model of a switch](image)
4.1 The Nominal Part of ARRs

Let \( C_a \rightarrow 0 \) and \( R_{sw} = R_D = R_{on} \). Assuming that all parameters take nominal values, then the nominal parts of two ARRs derived from the DBG take the following form.

\[
\begin{align*}
    r_1 &= -(R_L + R_{on})i_L - m_2u_C - \frac{L}{R} \frac{di_L}{dt} + E \quad (5) \\
    r_2 &= m_2i_L - C \frac{du_C}{dt} - \frac{1}{R} u_C \quad (6)
\end{align*}
\]

where \( m_2 = 1 - m_1 \).

The inductor current \( i_L \) and the voltage drop \( u_C \) across the capacitor in these ARRs are obtained either by computing a behavioural bond graph model of the boost converter in preferred integral causality or by measurement from the real system. That is, they are known.

4.2 The Uncertain Part of ARRs

By replacing the switch model as well as the other bond graph elements in the DBG by their incremental model gives the incremental DBG depicted in Fig. 8. In that bond graph, the elements \( De^* \) and \( Df^* \) denote virtual sensors of the uncertain part of the two residuals to be determined. If all parameters in the bond graph of Fig. 6 take nominal values, viz., there are no parameter variations, then variations of power variables with regard to any parameter changes vanish. In particular, \( \Delta i_L \) and \( \Delta u_C \) as inputs to the incremental DBG are equal to zero. This is indicated by the two sources of value zero in the upper part of the incremental DBG. By summing up incremental efforts at junction 1 and incremental flows at junction 0, the uncertain part of the two residuals is obtained.

\[
\begin{align*}
    \Delta r_1 &= \Delta u_C + \Delta u_L + \Delta u_a \\
    &= \delta L \frac{di_L}{dt} + \delta R_L R_L i_L + \delta R_{on} R_{on} i_L \quad (7) \\
    \Delta r_2 &= \Delta i_C + \Delta i_R - \Delta i_D \\
    &= \delta C \frac{du_C}{dt} - \frac{\delta R}{R} u_C \quad (8)
\end{align*}
\]

These results can be checked by deriving them from (5) and (6). If parameters in the latter equations are allowed to vary then

\[
\Delta r_2 = m_2 \frac{\Delta i_L}{L} - \left( \frac{C \frac{du_C}{dt}}{R} \right) - \left( \frac{1}{R} u_C \right) \quad (9)
\]

The variation of the second term reads

\[
\Delta C \frac{du_C}{dt} = (\Delta C) \frac{du_C}{dt} + C \frac{d}{dt} \frac{\Delta u_C}{R} \\
= \delta C \frac{du_C}{dt} + C \frac{d}{dt} \frac{\Delta u_C}{R} = 0 \quad (10)
\]

Furthermore,

\[
\Delta \left( \frac{1}{R} u_C \right) = \left( \frac{1}{R} \right) u_C + \frac{1}{R} \frac{\Delta u_C}{R} = 0 \\
= - \frac{1}{R^2} \Delta Ru_C = - \delta_R \frac{1}{R} u_C \quad (11)
\]

Substituting (11) and (10) into (9) gives (8). Likewise, (7) can be verified.

Now, adaptive thresholds for the uncertain part of the ARRs can be easily defined.

\[
|\Delta r_1| \leq |\delta L \frac{di_L}{dt}| + |\delta R_L R_L i_L| + |\delta R_{on} R_{on} i_L| =: th_1 \quad (12)
\]

and

\[
|\Delta r_2| \leq |\delta C \frac{du_C}{dt}| + |\delta R \frac{u_C}{R}| =: th_2 \quad (13)
\]

These thresholds can be used to ensure robust FDI with regard to parameter uncertainties. As long as noisy measurements that have passed through appropriate filtering give rise to residual values that are within these bounds no
alarm is reported to a diagnosis system. Moreover, as terms in the sum of absolute values, in general, are multiplied by a coefficient of value zero or one, these thresholds adapt to system modes. Thus, false alarms as well as overlooking faults due to bounds that are too wide with regard to a certain system mode can be avoided.

### 4.3 Numerical Computation of ARR Residuals and Adaptive Thresholds

For simulation, the nominal parameter values given in Tab. 1 are used. Parameter uncertainties $\delta_C$, $\delta_L$, $\delta_{RL}$, $\delta_R$ and $\delta_{R_{sw}}$ are assumed to be 2%. For simulating a fault scenario, the value of the load resistor is abruptly increased from 5.0 $\Omega$ to 50.0 $\Omega$ at $t = 0.05$ s. The resistor keeps this increased value for 0.01 s. Fig. 9 shows the time evolution of the voltage drop $u_C$ across the capacitor and its mean value. Fig. 10 displays the average time history of the two residuals $r_1$ and $r_2$. According to (5) and (6), residual $r_2$ is sensitive to this parameter fault while $r_1$ is not.

An enlarged plot of the time history of the bounds of residual $r_2$ computed according to (8) clearly shows that the bounds are adaptive and that the values of $r_2$ are within these bounds (cf. Fig. 11).

---

**Table 1. Parameters of the boost converter circuit**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>12.0</td>
<td>V</td>
<td>Voltage supply</td>
</tr>
<tr>
<td>L</td>
<td>1.0</td>
<td>mH</td>
<td>Inductance</td>
</tr>
<tr>
<td>$R_L$</td>
<td>0.1</td>
<td>$\Omega$</td>
<td>Resistance of the coil</td>
</tr>
<tr>
<td>C</td>
<td>500</td>
<td>$\mu F$</td>
<td>Capacitance</td>
</tr>
<tr>
<td>R</td>
<td>5.0</td>
<td>$\Omega$</td>
<td>Load resistance</td>
</tr>
<tr>
<td>$T_s$</td>
<td>$10^{-3}$</td>
<td>s</td>
<td>Switching time period</td>
</tr>
<tr>
<td>$d_1$</td>
<td>0.45</td>
<td></td>
<td>Duty ratio</td>
</tr>
</tbody>
</table>

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**Fig. 8. Incremental diagnostic bond graph of the boost converter circuit**

**Fig. 9. Time evolution of the voltage drop $u_C$ across the capacitor**

**Fig. 10. Time evolution of residuals $r_1$ and $r_2$**
models are the following. The results of this approach to FDI in hybrid system
tive to a change of the load resistor. Finally, Fig. 12 displays the bounds of residual

\[ r_t(\text{time}[s]) \]

Fig. 11. Time history of the averaged residual \( r_2 \) and its thresholds

\[ r_t(\text{time}[s]) \]

Fig. 12. Time history of the residual \( r_1 \) and its thresholds

In an offline simulation, a behavioural process model is used instead of the real process. That means, the time derivatives of variables in the expressions for the variations of the residuals are outputs of the process model and can be expressed by its state variables. There is no need to perform discrete differentiation of measured variables from the real process. For instance, \( \frac{dC_{duC}}{dt} \) in (8) can be expressed by means of (14) obtained from the behavioural bond graph of the boost converter in Fig. 6.

\[ C_{duC} \frac{dC_{duC}}{dt} = m_2i_L - Ru_C \]  (14)

Finally, Fig. 12 displays the bounds of residual \( r_1 \) insensitive to a change of the load resistor.

5. CONCLUSION

The results of this approach to FDI in hybrid system models are the following:

- As switching devices are modelled by means of a Boolean controlled transformer and a resistor with fixed conductance causality accounting for the ON-resistance, one unique set of ARRs for all system modes can be derived from a DBG.
- The nominal part of ARRs is obtained from a DBG with nominal parameters.
- An incremental diagnostic bond graph can be systematically constructed from an original diagnostic bond graph. Bond graph elements are replaced by their incremental model.
- The uncertain part of ARR residuals with regard to parameter uncertainties can be obtained from an incremental diagnostic bond graph by adding efforts or flows respectively at junctions. The resulting variations of ARR residuals are used to define adaptive thresholds. They are a sum of terms of absolute value. In general, some terms are multiplied by the modulus of a transformer controlled by a Boolean variable so that the thresholds are system mode dependent.

The obtained adaptive thresholds can be used to achieve robust FDI with regard to parameter uncertainties in order to avoid false alarms or disregarding faults. The paper shows that incremental bond graphs can be also used for this task.

REFERENCES


